Appendix D – Digital Modulation and GMSK

A brief introduction to digital modulation schemes is given, showing the logical development of GMSK from simpler schemes. GMSK is of interest since it is used in the GSM system. The phase and amplitude relations between carrier cycles over a data bit are developed, enabling rigourous modelling of ensemble fields to be carried out.

D1. <u>Phase shift keying</u>

For binary PSK (BPSK)

$S_0(t) = A \cos(\omega t)$ represents binary "0"

$S_1(t) = A \cos(\omega t + \pi)$ represents binary "1"

For M-ary PSK, M different phases are required, and every n (where $M=2^n$) bits of the binary bit stream are coded as one signal that is transmitted as

$$A \sin(\omega t + \theta_j)$$

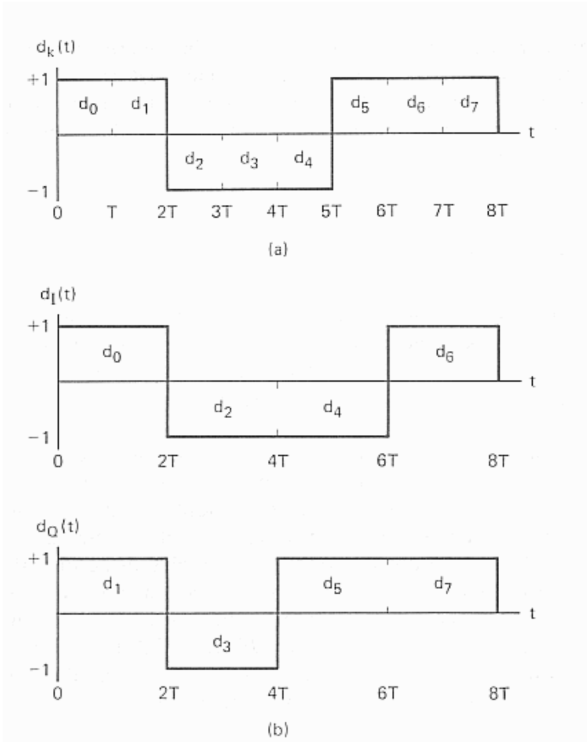
j=1,...,M.

D2. <u>Quadrature Phase Shift Keying</u>

If we define four signals, each with a phase shift differing by 90^0 then we have quadrature phase shift keying (QPSK).

The input binary bit stream $\{d_k\}$, d_k = 0,1,2,..... arrives at the modulator input at a rate 1/T bits/sec and is separated into two data streams $d_I(t)$ and $d_Q(t)$ containing odd and even bits respectively.

$$d_I(t) = d_0, d_2, d_4, ...$$
$$d_Q(t) = d_1, d_3, d_5, ...$$



A convenient orthogonal realisation of a QPSK waveform , s(t) is achieved by amplitude modulating the in-phase and quadrature data
streams onto the cosine and sine functions of a carrier wave as follows:

$$s(t)=1/\sqrt{2}\, d_I(t) \cos(2\pi ft + \pi/4) + 1/\sqrt{2}\, d_Q(t) \sin(2\pi ft + \pi/4)$$

Using trigonometric identities this can also be written as

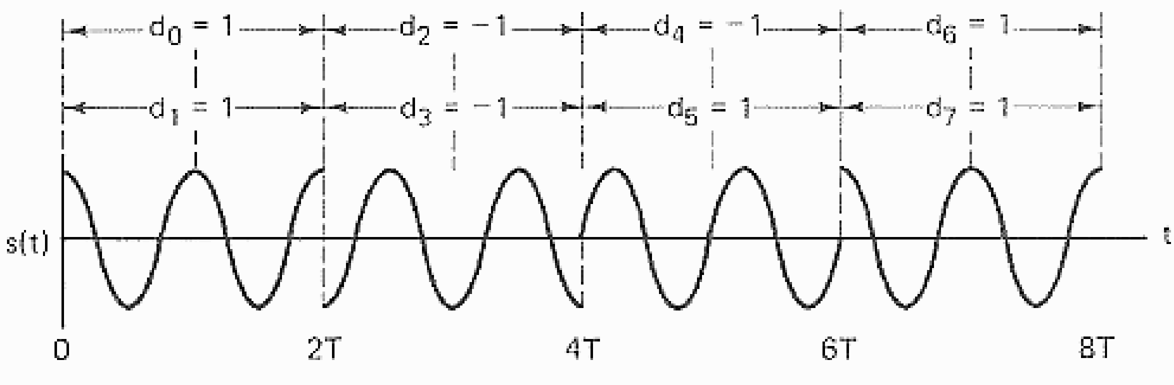
$$s(t)=A \cos[2\pi ft + \pi/4 + \theta(t)].$$

The pulse stream $d_I(t)$ modulates the cosine function with an amplitude of ± 1. This is equivalent to shifting the phase of the cosine function by 0 or π; consequently this produces a BPSK waveform. Similarly the pulse stream $d_Q(t)$ modulates the sine function, yielding a BPSK waveform orthogonal to the cosine function. The summation of these two orthogonal waveforms is the QPSK waveform.

The values of $\theta(t) = 0, -(\pi/2), \pi/2, \pi$ represent the four possible combinations of $a_I(t)$ and $a_Q(t)$.

Each of the four possible phases of carriers represents two bits of data. Thus there are two bits per symbol. Since the symbol rate for QPSK is half the bit rate, twice as much data can be carried in the same amount of channel bandwidth as compared to BPSK. This is possible because the two signals I and Q are orthogonal to each other and can be transmitted without interfering with each other.

In QPSK the carrier phase can change only once every 2T secs. If from one T interval to the next one, neither bit stream changes sign, the carrier phase remains unchanged. If one component $a_I(t)$ or $a_Q(t)$ changes sign, a phase change of $\pi/2$ occurs. However if both components change sign then a phase shift of π occurs.

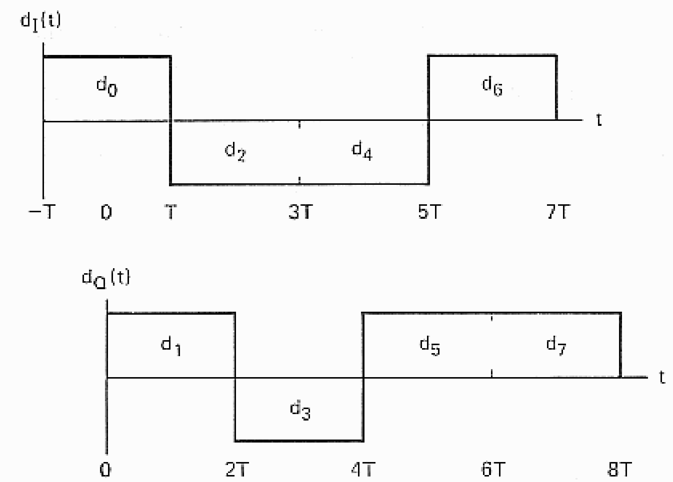


(a) QPSK

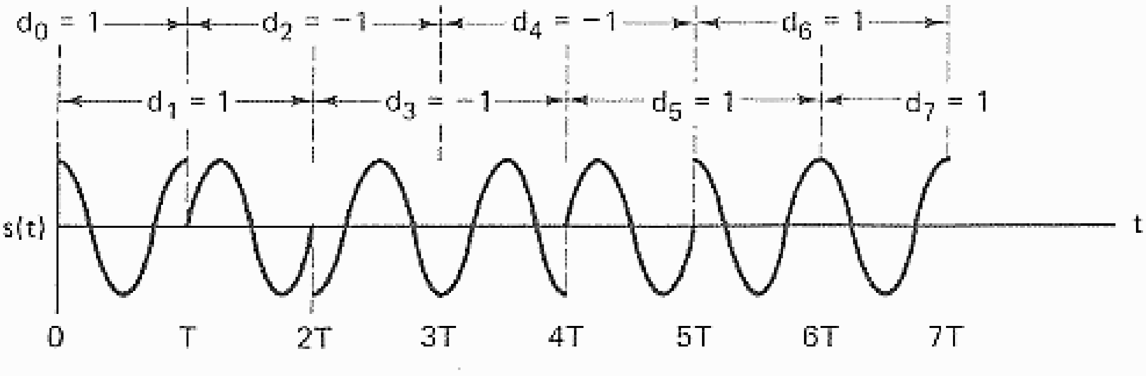
If a QPSK modulated signal undergoes filtering to reduce the spectral side lobes, the resulting waveform will no longer have a constant envelop and in fact, the occasional 180^o shifts in phase will cause the envelope to go to zero momentarily.

D3. Offset Quadrature Phase Shift Keying

If the two bit streams I and Q are offset by a 1/2 bit interval, then the amplitude fluctuations are minimised since the phase never changes by 180^o. This modulation scheme, Offset Quadrature Phase shift Keying (OQPSK) is obtained from QPSK by delaying the odd bit stream by half a bit interval with respect to the even bit stream.



Thus the range of phase transitions is 0^o and 90^o (the possibility of a phase shift of 180^o is eliminated) and occurs twice as often, but with half the intensity of the QPSK. While amplitude fluctuations still occur in the transmitter and receiver they have smaller magnitude. The bit error rate for QPSK and OQPSK are the same as for BPSK.



(b) OQPSK

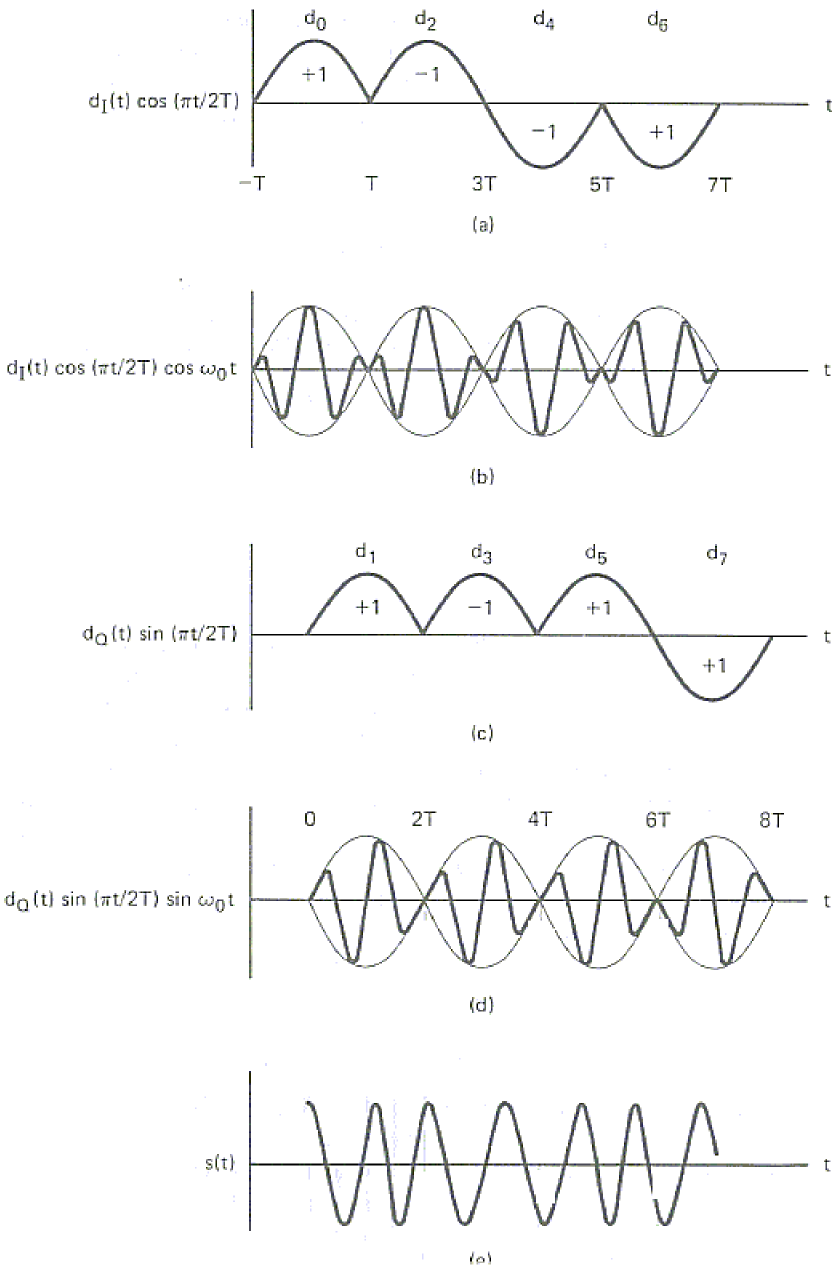
When an OQPSK signal undergoes bandlimiting, the resulting intersymbol interference causes the envelop to droop slightly to the region of $\pm 90^o$ phase transition, but since the phase transitions of 180 have been avoided in OQPSK, the envelop will never go to zero as it does in QPSK.

D4. <u>Minimum Shift Keying</u>

We previously showed that OQPSK is obtained from QPSK by delaying the Q data stream by 1 bit or T seconds with respect to the I data stream. This delay has no effect on the error or bandwidth.

Minimum Shift Keying (MSK) is derived from OQPSK by replacing the rectangular pulse in amplitude with a half-cycle sinusoidal pulse. The MSK signal is defined as:

$S(t) = d(t) \cos(\pi t/2T) \cos 2\pi ft + d(t) \sin(\pi t/2T) \sin 2\pi ft.$



The MSK modulation makes the phase change linear and limited to ± (π/2) over a bit interval T. This enables MSK to provide a significant improvement over QPSK. Because of the effect of the linear phase change, the power spectral density has low side lobes that help to control adjacent-channel interference. However the main lobe becomes wider than the quadrature shift keying.

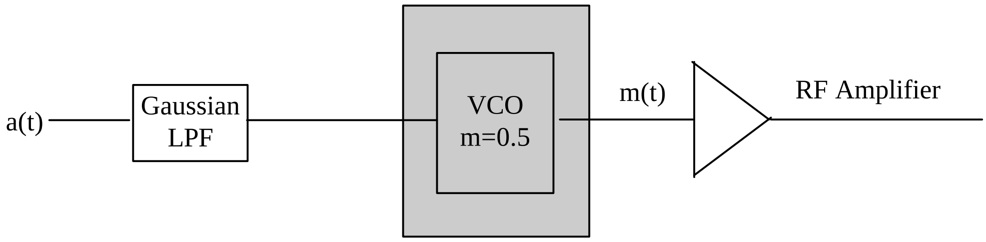
D5. Gausssian Minimum Shift Keying (GMSK)

In MSK we replace the rectangular pulse with a sinusoidal pulse. Obviously other pulse shapes are possible. A Gaussian-shaped impulse response filter generates a signal with low side lobes and narrower main lobe than the rectangular pulse. Since the filter theoretically has output before input, it can only be approximated by a delayed and shaped impulse response that has a Gaussian - like shape. This modulation is called Gaussian Minimum Shift Keying (GMSK).

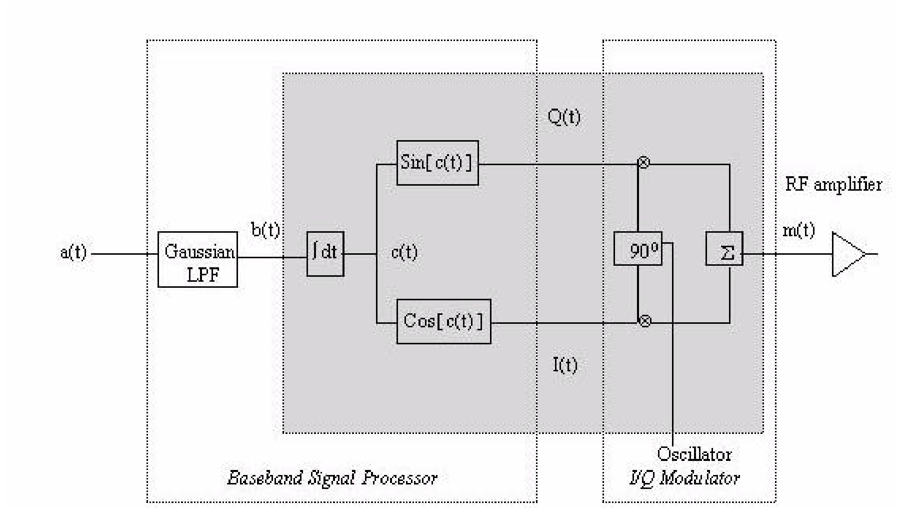
The relationship between the premodulation filter bandwidth, B and the bit period, T defines the bandwidth of the system. GSM designers used a BT = 0.3 with a channel data rate of 270.8 kbs. This compromises between a bit error rate and an out-of-band interference since the narrow filter increases the intersymbol interference and reduces the signal power.

D5.1 GMSK Modulation

There are two methods to generate GMSK, one is frequency shift keyed modulation, the other is quadrature phase shift keyed modulation.



GMSK implemented by Frequency Shift Keying modulation with FM-VCO.



GMSK implemented by a quadrature baseband method.

The shaded areas in the two above figures have the same function.

The GMSK VCO-modulator architecture as shown in the first is simple but is not however, suitable for coherent demodulation due to component tolerance problems. This method requires that the frequency deviation factor of the VCO exactly equals 0.5, but the modulation index of conventional VCO based transmitters drifts over time and temperature.

The implementation in the second employs a quadrature baseband process followed by a quadrature modulator. With this implementation, the modulation index can be maintained at exactly 0.5. This method is also cheaper to implement.

Both methods lead to the same GMSK modulated signal.

We are going to be looking at the second of these two methods, that is we shall be looking at a quadrature baseband processor followed by a quadrature modulator as shown in the second.

The Gaussian low-pass filter has an impulse response given by the following equation

$$g(t) = \frac{1}{2T}\left[Q\left(2\pi B_b \frac{t - T/2}{\sqrt{\ln 2}}\right) - Q\left(2\pi B_b \frac{t + T/2}{\sqrt{\ln 2}}\right)\right]$$

for

$$0 \le B_b T \le \infty$$

where Q(t) is the Q-function

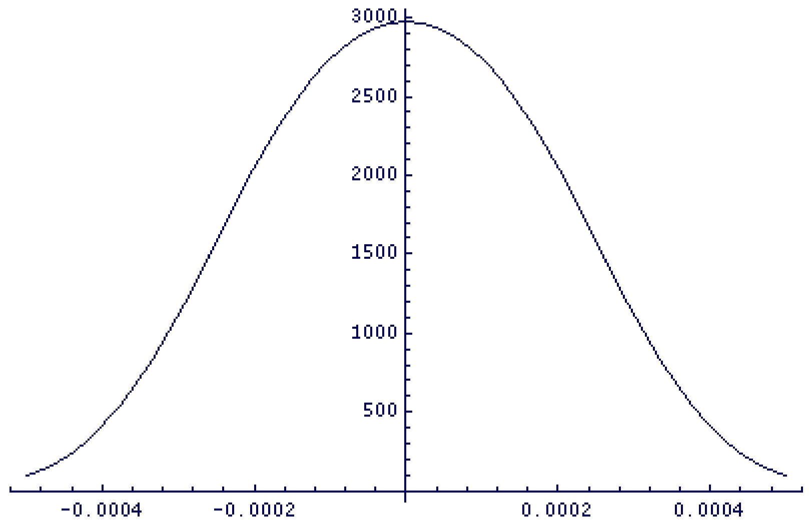
$$Q(t) = \int_t \frac{1}{\sqrt{2}} \exp(-x^2/2)\, dx \,,$$

B_b is the bandwidth of the low pass filter having a Gaussian shaped spectrum, T is the bit period and $BN=B_b T$ is the normalised bandwidth.

To demonstrate this, we are looking at a filter with a bandwidth of $B_b = 1000$ and a bit rate of $T=1/2000$, i.e. a normalised bandwidth $B_N=B_b.T = 0.5$.

The impulse response of the Gaussian low-pass filter has to be truncated and scaled, according to the B_N value, to ensure that the effect of a single 1 passing through the filter, is a phases change of $\pi/2$.

For a B_N of 0.5 the filter response is truncated, symmetrically around zero, to two bit periods, i.e. from -T to T. The truncated filter response is represented graphically in the following figure.



The truncated and scaled impulse response of the Gaussian low-pass filter.

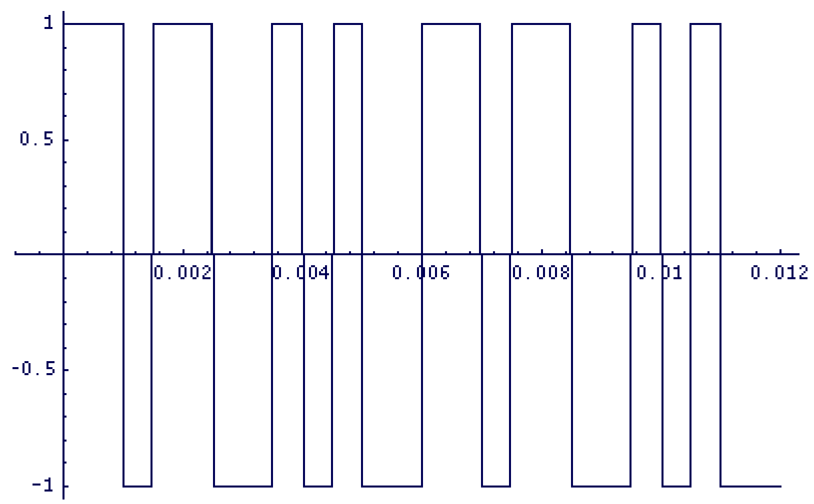
Ensuring that the response of the filter to a single 1 is a phase change of π/2, is equivalent to choosing the constant *K* to satisfy the following equation

$$\int_{-T}^{T} Kg(t)dt = \pi/2.$$

To demonstrate the modulation, we are using the following randomly chosen binary data stream. (This data stream repeats after 12 bits.)

{1,1,-1,1,1,-1,-1,1,-1,1,-1,-1, 1,1,-1,1,1,-1,-1,1,-1,1,-1,-1,............}.

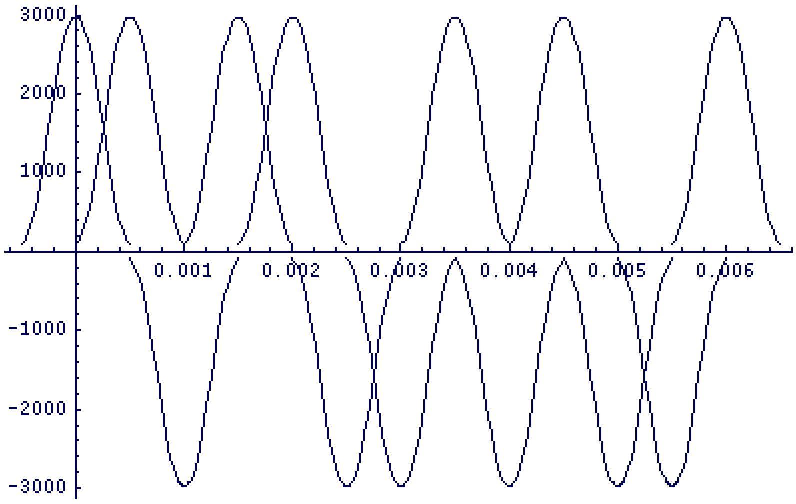
The beginning of this data stream can be represented graphically by the following



The beginning of the data stream being sent through the filter.

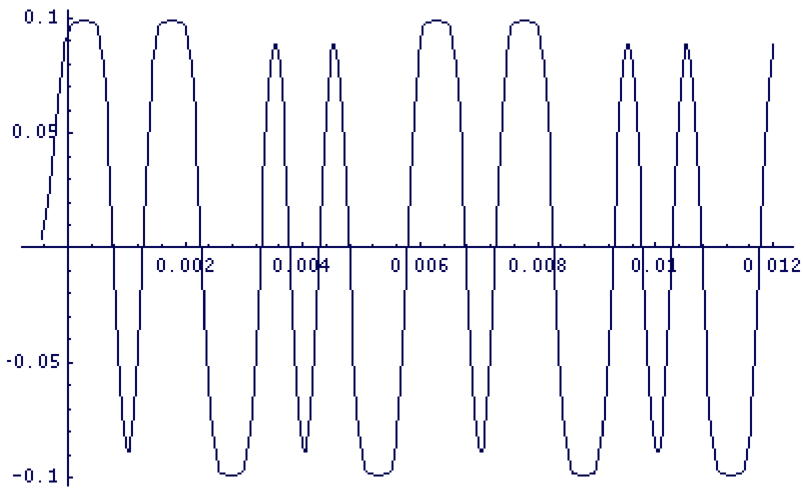
As the data passes through the filter it is shaped and ISI (inter symbol interference) is introduced since more than one bit is passing through the filter at any one time. For $B_N = 0.5$, since the bits are spread over two bit periods, the second bit enters the filter as the first is half way through, the third enters as the first leaves etc....

The first few Gaussian shaped pulses are represented graphically in the following figure.



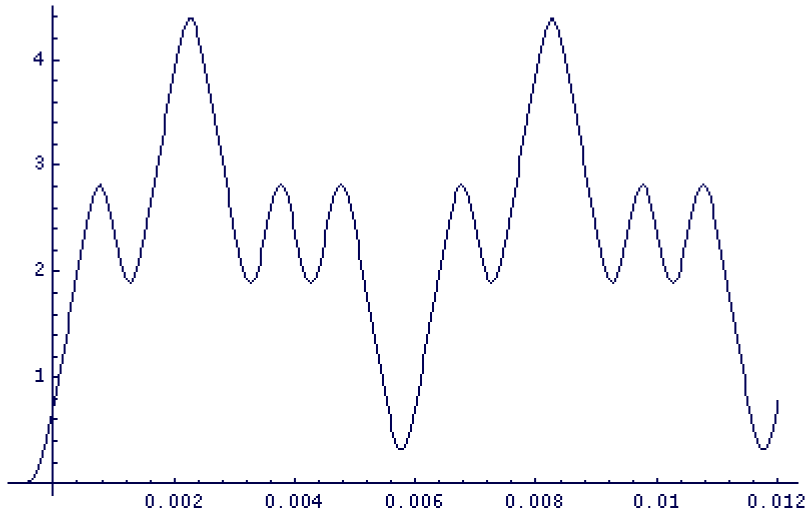
The individual shaped pulses representing the data stream.

These individual shaped pulses are then added together to give a function which is represented graphically in the following figure. This is the function denoted by b(t).



The function b(t) as in the second figure

This function, b(t), is then integrated, with respect to t (time) from t to ∞, to give the function c(t) as shown in the second figure. This function c(t) is represented graphically below.

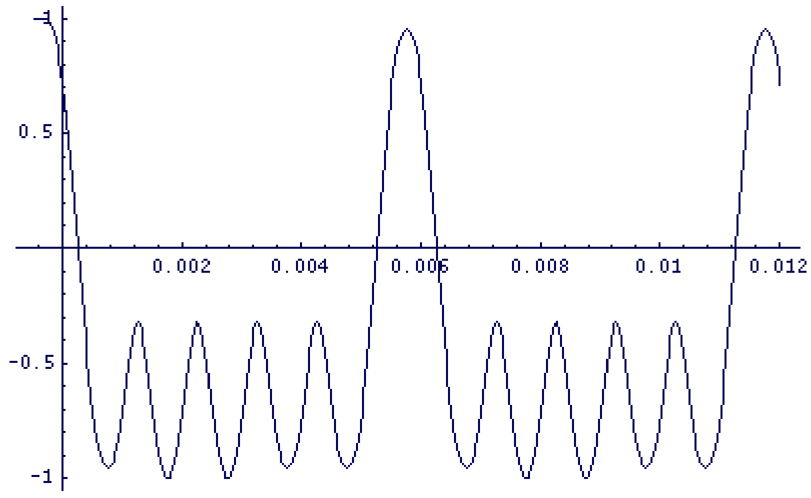


The function c(t) as in the second figure.

Once we have the function c(t), we take Sine and Cosine functions of it to produce the I and Q-baseband signals. Taking the Cosine of c(t) produces the I-baseband signal I(t) i.e.

$$I(t) = \text{Cos}[\ c(t)\ ].$$

This function I(t) is represented graphically below.

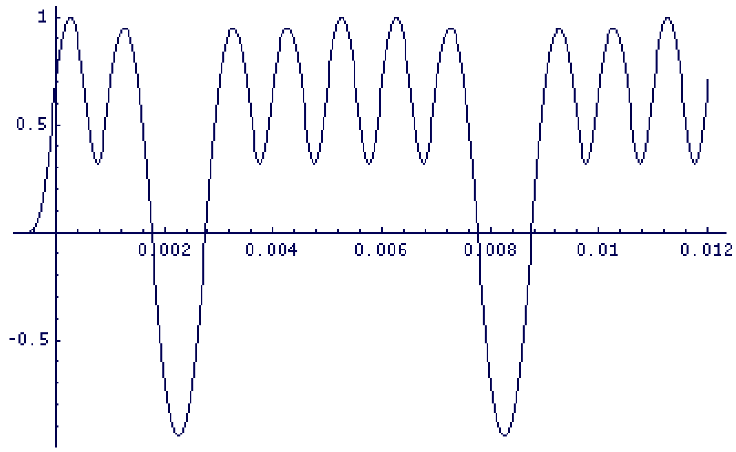


The I-baseband signal, i.e. the function I(t) as the second figure

Taking the Sine of c(t) produces the Q-baseband signal Q(t) i.e.

$$Q(t) = \mathrm{Sin}[\ c(t)\ ].$$

This function Q(t) is represented graphically below.



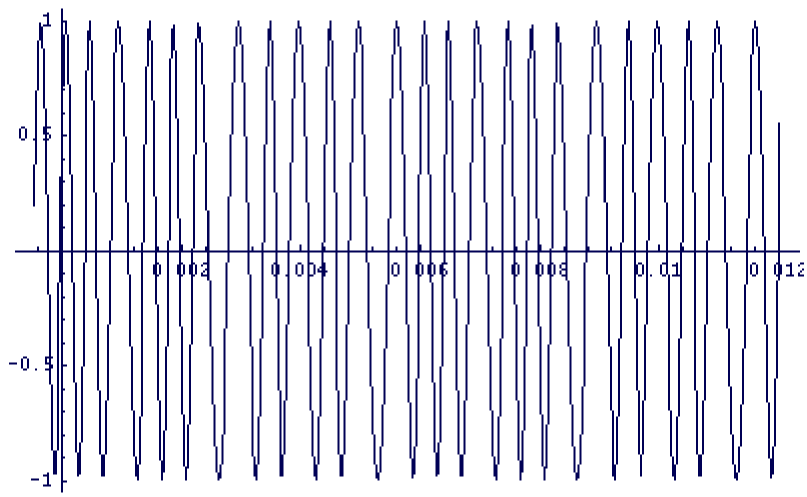
The Q-baseband signal, i.e. the function Q(t) as in the second figure.

These two functions I(t) and Q(t) are then passed through the I/Q modulator which leads to the output signal m(t) which can be written as

$$m(t) = \mathrm{Sin}(2\pi f_c\ t)\ I(t) + \mathrm{Cos}(2\pi f_c t)\ Q(t),$$

where f_c is the carrier frequency used as the oscillator in the second figure

The GMSK signal m(t) is represented



The GMSK modulated signal m(t) as in the second figure.